A Practical Technique for Designing Multiport Coupling Networks

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Abstract—**A new technique is proposed for designing a passive lossless coupling network transforming any prescribed N by N symmetric immittance matrix into a corresponding N by N diagonal immittance matrix. A principal application of the technique is in the design of matching networks between N uncoupled resistive source impedances and planar antenna arrays. The technique is based upon repeated applications of Givens rotations [1], which can be implemented by a cascade connection of four-port directional couplers. Thus, both in the design technique and in the subsequent hardware implementation, our approach represents a significant departure from past design procedures. Existing synthesis methods involve the use of multiwinding transformers, which are impractical at microwave frequencies.**

I. Introduction

THE realization of N-port immittance matrices was solved by Carlin with ideal multiwinding transformers in 1955 [2]. Later, Youla extended the method to systems described by S-matrices [3]. In both cases the implementation required using ideal multiwinding transformers, which are impractical as microwave components. In this paper, an alternative approach is proposed based upon repeated applications of Givens rotations. The subsequent implementation is made with cascaded four-port directional couplers. The motivation for designing such networks is to optimize antenna array feeds with respect to impedance matching, bandwidth, and pattern constraints such as directivity and sidelobe levels [4], [5].

II. Antenna Arrays

Reactively loaded antenna arrays have been analyzed in the literature; however, the loading has been restricted to special cases. Particularly, the case of isolated loads attached across each antenna port, together with either a parallel or a series transmission line feed, have received extensive treatment [4], [5]. Such topologies are not sufficiently general to enable a designer to handle complex loads associated with a full $N \times N$ symmetric immittance matrix. The technique described in this paper, on the other hand, provides a general design procedure for realizing the full multiport feed network in order to optimize array performance.

In the general case, the feed network for an N-element array consists of a lossless interconnecting network with N ports attached to the antenna elements and N ports terminated in uncoupled source resistances, as shown in Fig. 1.

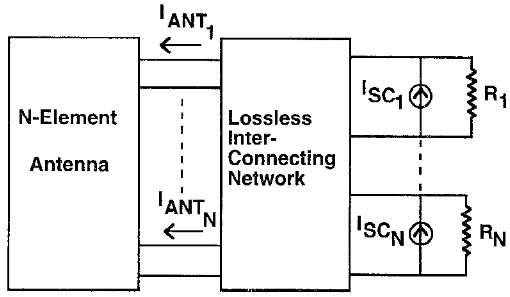

Fig. 1. General antenna feed configuration.

Fig. 1 also shows the relationship between the source current, I_{sc}, and the current vector at the antenna ports, I_{ant}. (The latter defines the array aperture distribution, assuming a monopole array.) Since I_{sc} and I_{ant} are related through network equilibrium equations, one may solve for the required source currents to provide the desired phase and amplitude taper over the array aperture.

An obvious application of this technique to an array of moderate size is illustrated in Fig. 2, i.e., a conjugate match to the antenna impedance.

The coupling networks, A and B, are lossless multiport networks that transform the isolated reactive and resistive series elements into the indicated load impedance at the antenna. As will be shown, the feed network in Fig. 2 yields an impedance matrix Z_{in} at the sources that is real, diagonal, and matched to the impedances R_i. Hence, the match at center frequency (where $Z_{LOAD} = Z^*_{ANT}$) is ideal and independent of aperture distribution.

III. Design Procedure

A. Design Algorithm

The design to be introduced leads to coupling structures composed of reactances and multiport components described by S-matrices of the form

$$S_a = \begin{pmatrix} 0 & A \\ A^t & 0 \end{pmatrix} \tag{1}$$

where A is a real N by N matrix whose columns are orthogonal vectors and A^t is its transpose. As a preliminary to the design procedure, we present an important property about networks whose S-matrices are of this form. This property

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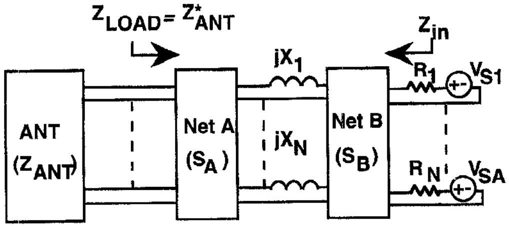

Fig. 2. Terminated conjugate match feed network.

is central to our design procedure and referred to below as Theorem 1.

Theorem 1: The $N \times N$ scattering matrix, S_{in}, of a $2N$ port network, described by S_A as in (1), and terminated with an N port network, described by S' as shown in Fig. 3, can be expressed as

$$S_{in} = AS'A^t. \tag{2}$$

Furthermore, if $A = (\vec{e}_1 \cdots \vec{e}_n)$ is an $N \times N$ rectangular matrix composed of an orthonormal set of real vectors, then the corresponding impedance matrices are related by

$$Z_{in} = AZ'A^t. \tag{3}$$

A proof of Theorem 1 is readily obtained by following the discussion in Newcomb [6].

At this point it should be mentioned that the time delay of wave propagation at microwave frequencies is not negligible, and that S-matrices of practical networks include a multiplicative phase term, $e^{-j\phi}$. The familiar scalar expression for translation of a terminating impedance through a transmission line of electrical length ϕ can be expressed as a matrix product directly. The normalized impedance seen at the input of the line is expressed in terms of the termination as

$$Z_{\text{in}} = (Z_{\text{term}} \cos\phi + jI \sin\phi)(I\cos\phi + jZ_{\text{term}} \sin\phi)^{-1}$$

where

$$Z_{\text{term}} = AZ'A^t. \tag{4}$$

Thus, if an S-matrix in the form of purely real elements (corresponding to electrical lengths of $\phi = 0$), then (3) follows. On the other hand, if the S-matrix has purely imaginary elements, (corresponding to $\phi = \frac{\pi}{2}$), then (4) takes the form

$$Z_{\text{in}} = AY'A^t \tag{5}$$

where $Y' = Z'^{-1}$. In the paper, we will use (3), although (5) could be used for a dual derivation.

The design procedure can now be formulated.

Consider a specified $N \times N$ positive real, symmetric impedance matrix $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$, as shown in Fig. 4. The design objective is to implement the coupling network between the resistive terminations $R_1 \cdots R_N$ and Z_{in}. Since X_{in} is real and symmetric, it may be diagonalized by an orthogonal matrix A, where the columns are the N eigenvectors of X_{in}. The matrix A is used (as indicated in (1)) to design the coupling network A in Fig. 4. From (3), $R_{\text{in}} + jX_{\text{in}} = AZ'A^t$, where Z' is the impedance matrix of the network inside the dotted lines. Equating imaginary parts and solving for X' (the imaginary part of Z') yields $X' = A^t X_{\text{in}} A$. X' is a diagonal matrix corresponding to the series reactive elements of Fig. 4.

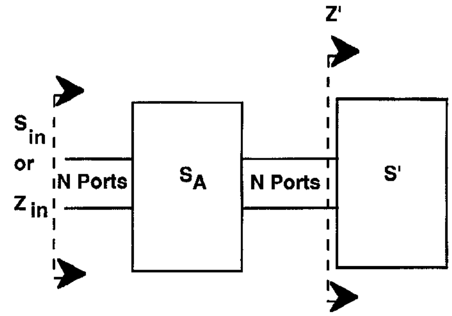

Fig. 3. A cascade connection, pertaining to Theorem 1.

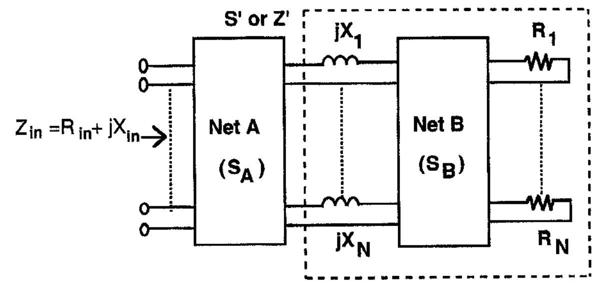

Fig. 4. Definitions used in the design procedure.

R', the real part of Z', is determined as $R' = A^t R_{\text{in}} A$. This matrix is positive real and symmetric. The matrix can be diagonalized by a second orthogonal matrix B, where the columns of B are the eigenvectors of R'. The matrix B is used to design the coupling network B of Fig. 4. The resistances terminating Network B are the eigenvalues of R' corresponding to the eigenvectors, which make up the columns of B.

So far the derivation follows an existing design procedure [2] that realizes the coupling network with multiwinding transformers (impractical at microwave frequencies). Our novel approach utilizes repeated applications of Givens rotations as shown in the Appendix. The Appendix furthermore shows how these rotations can be implemented by at most $N(N-1)/2$ cascaded four-port directional couplers.

Our technique thus represents a new, practical approach to the design of lossless matching networks at microwave frequencies. To our knowledge, no other procedure for designing practical coupling networks matching arbitrary N by N admittance matrices to diagonal admittance matrices at microwave frequencies has been documented in the literature.

The design procedure for realizing an arbitrary passive impedance matrix $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$ may thus be summarized as follows:

1) Compute the eigenvalues and eigenvectors of X_{in}.
2) Design the transformation network whose scattering matrix S_A and series reactive elements X_i correspond to the eigenvectors and eigenvalues of X_{in}.
3) Compute the eigenvalues and eigenvectors of $R' = A^t R_{\text{in}} A$.
4) Design the transformation network whose scattering matrix is S_B and resistive elements, R_i corresponding to the eigenvectors and eigenvalues of R'.

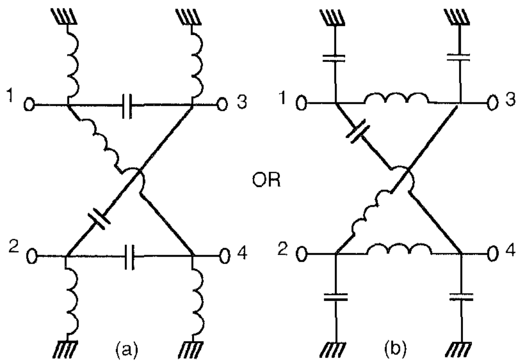

Fig. 5 Four-port directional couplers

B. Design of Lumped-Element Four-Port Directional Couplers

In order to implement the design, one needs directional couplers with an S-matrix of the form

$$S = \begin{pmatrix} 0 & A \\ A^t & 0 \end{pmatrix} \quad \text{with} \quad A = \begin{pmatrix} S_{13} & S_{14} \\ S_{23} & S_{24} \end{pmatrix}. \tag{6}$$

Lumped-element directional couplers with purely imaginary A-matrices have been shown in [7] to have the topology of Fig. 5.

A cascade connection of the two networks shown in Fig. 5 implements S-matrices of the form of (6) in which A is purely real. To see this, consider S-matrices of the form of (6) describing lossless networks. The constraints on the phases of the scattering coefficients of such networks are

$$2\phi_{13} = \phi_{14} + \phi_{23} + (2k+1)\pi. \quad k \text{ any integer.} \tag{7}$$

If, in addition, the circuit parameters are constrained to insure $S_{23} = -S_{14}$, then

$$\phi_{13} = \phi_{14} + k\pi$$

and

$$A = e^{j\phi_{14}} \begin{pmatrix} \sqrt{1-|S_{14}|^2} & (-1)^k|S_{14}| \\ -(-1)^k|S_{14}| & \sqrt{1-|S_{14}|^2} \end{pmatrix}. \tag{8}$$

For the networks shown in Fig. 5, $S_{14} = -jb_1$ where b_1 is the normalized susceptance linking ports 1 and 4. Using Fig. 5(a) the A-matrix becomes

$$A_a = \frac{1}{j} \begin{pmatrix} \cos(\theta_1) & \sin(\theta_1) \\ -\sin(\theta_1) & \cos(\theta_1) \end{pmatrix}.$$

For Fig. 5(b), the A-matrix becomes

$$A_b = j \begin{pmatrix} \cos(\theta_2) & \sin(\theta_2) \\ -\sin(\theta_2) & \cos(\theta_2) \end{pmatrix}.$$

Cascading the two circuits yields the desired S-matrix

$$S_{ab} = \begin{pmatrix} 0 & A_{ab} \\ A_{ab}^t & 0 \end{pmatrix}$$

with

$$A_{ab} = \begin{pmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix}.$$

Hence, one may construct lumped-element four-port directional couplers having the form of (6) and thereby build up the S-matrix needed to transform the terminating impedances. Although the reason for cascading the two circuits was to provide a real S-matrix, the resulting circuit has an extra branch whose additional degrees of freedom may be used to increase the bandwidth [7].

IV. Design Example of Coupling Network for a Four-Element Linear Array

In order to illustrate the realization technique with a meaningful example, a network will be designed that provides a conjugate match to a specified antenna admittance matrix. For the antenna array, we choose four slots of 0.7λ spacing oriented broadside to the axis of the array. For a stripline-fed cavity-backed slot, one can adjust the feed-point to match the feedline characteristic impedance to the slot impedance at resonance. This is equivalent to coupling the 4×4 antenna impedance matrix to the four feed lines through a wideband transformer.

Assuming each slot has an identical aperture source distribution, the admittance matrix has Toeplitz symmetry, i.e., the elements of the admittance matrix have the form: $Y_{ij} = Y_{i+1\ j+1}$ for $i, j < N-1$ and all self-terms are identical. The inverse of the Toeplitz (symmetric) matrix is centrosymmetric, i.e., it is symmetric about the main and cross diagonals (a persymmetric matrix has only the latter symmetry). Centrosymmetry reduces the number and complexity of directional couplers required to diagonalize the impedance matrix.

The conjugate load that matches the four slot antenna at the center frequency of 3 GHz is described by the following Toeplitz matrix normalized to the characteristic admittance of the feed line

$$Y_{\text{in}} = \begin{bmatrix} 1.0000 & -.3400 & .0820 & .0820 \\ -.3400 & 1.0000 & -.3400 & .0820 \\ .0820 & -.3400 & 1.0000 & -.3400 \\ .0820 & .0820 & -.3400 & 1.0000 \end{bmatrix} + j \begin{bmatrix} 0 & .0060 & .1600 & -.0900 \\ .0060 & 0 & .0060 & .1600 \\ .1600 & .0060 & 0 & .0060 \\ -.0900 & .1600 & .0060 & 0 \end{bmatrix}. \tag{9}$$

Since our design procedure for realizing this load is in terms of Z_{in}, Y_{in} must first be inverted, producing the normalized impedance matrix

$$Z_{\text{in}} = \begin{bmatrix} 1.1099 & .3796 & -.0105 & -.1349 \\ .3796 & 1.2214 & .3698 & -.0105 \\ -.0105 & .3698 & 1.2214 & .3796 \\ -.1349 & -.0105 & .3796 & 1.1099 \end{bmatrix} + j \begin{bmatrix} -.0134 & -.0575 & -.2060 & -.0250 \\ -.0575 & -.0575 & -.1529 & -.2060 \\ -.2060 & -.1529 & -.0575 & -.0575 \\ -.0250 & -.2060 & -.0575 & -.0134 \end{bmatrix}. \tag{10}$$

The four step design procedure yields the following.

Step 1: Coupling network A, described by S_A, can be computed from an eigenvalue analysis of the reactive part of the impedance matrix. It should have an A matrix of the eigenvectors of the imaginary part of Z_{in} as shown in (1)

$$A = \begin{bmatrix} 0.4153 & 0.5638 & -0.5723 & 0.4268 \\ 0.5723 & -0.4268 & 0.4153 & 0.5638 \\ 0.5723 & 0.4268 & 0.4153 & -0.5638 \\ 0.4153 & -0.5638 & -0.5723 & -0.4268 \end{bmatrix}. \quad (11)$$

The series reactances between network A and network B are given by the eigenvalues

$$\begin{aligned} X_1 &= -.4016 \\ X_2 &= -.1008 \\ X_3 &= +.1529 \\ X_4 &= +.2078. \end{aligned} \quad (12)$$

Step 2: The scattering matrix S_A is readily obtained from (6), generating

$$S_A = \begin{bmatrix} 0 & A \\ A^t & 0 \end{bmatrix}. \quad (13)$$

In order to design network A, the upper right-hand corner of this matrix must be factored into six matrices. Using the method of the Appendix the result is

$$\begin{aligned} A &= \begin{bmatrix} 0.4153 & 0.5638 & -0.5723 & 0.4268 \\ 0.5723 & -0.4268 & 0.4153 & 0.5638 \\ 0.5723 & 0.4268 & 0.4153 & -0.5638 \\ 0.4153 & -0.5638 & -0.5723 & -0.4268 \end{bmatrix} \\ &= \begin{bmatrix} 0.5873 & -0.8094 & 0 & 0 \\ 0.8094 & 0.5873 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\ &\times \begin{bmatrix} 0.7773 & 0 & -0.6291 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0.6291 & 0 & 0.7773 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\ &\times \begin{bmatrix} 0.9097 & 0 & 0 & -0.4153 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0.4153 & 0 & 0 & 0.9097 \end{bmatrix} \\ &\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & -0.9008 & -0.4342 & 0 \\ 0 & 0.4342 & -0.9008 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\ &\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0.7848 & 0 & 0.6198 \\ 0 & 0 & 1.0000 & 0 \\ 0 & -0.6198 & 0 & 0.7848 \end{bmatrix} \\ &\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & -0.5978 & 0.8016 \\ 0 & 0 & -0.8016 & -0.5978 \end{bmatrix}. \end{aligned} \quad (14)$$

Each of the six matrices can be realized with one four-port directional coupler and two straight through connections. For example, the scattering matrix of the network corresponding to the second factor on the right-hand side of (14) is as shown in (15) at the bottom of the page. The network realization of the above scattering matrix is shown in Fig. 6.

Six such networks, cascaded, make up the network A of Fig. 4.

Step 3: After network A is realized, the real part of the input impedance can be realized by solving for the eigenvalues and eigenvectors of

$$A^t R_{in} A = \begin{bmatrix} 1.7295 & 0.0000 & 0.1785 & 0.0000 \\ 0.0000 & 0.7262 & 0.0000 & 0.2951 \\ 0.1785 & 0.0000 & 0.8367 & 0.0000 \\ 0.0000 & 0.2951 & 0.0000 & 1.3702 \end{bmatrix}. \quad (16)$$

The matrix has many zero elements because of the centrosymmetry of the original impedance matrix. The upper right-hand corner of the scattering matrix for network B is given by the eigenvectors of the matrix in (16). The result is

$$B = \begin{bmatrix} -0.9820 & 0.1890 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.3625 & -0.9320 \\ -0.1890 & -0.9820 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & -0.9320 & 0.3625 \end{bmatrix}. \quad (17)$$

Step 4: The terminating resistances of the coupling network B are specified by the following eigenvalues

$$\begin{aligned} R_1 &= 1.7639 \\ R_2 &= .8023 \\ R_3 &= 1.4850 \\ R_4 &= .6114. \end{aligned} \quad (18)$$

The final realization is shown in Fig. 7. The matrix in (17) is factored to find the division ratios for the directional couplers. Following the procedure of the Appendix leads to the following six rotation matrices, whose product equals B

$$B = \begin{bmatrix} -1.0000 & 0 & 0 & 0 \\ 0 & -1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0.7773 & 0 & -0.6291 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6291 & 0 & 0.7773 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\ 0.7773 & 0 & 0.6291 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.6291 & 0 & 0.7773 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

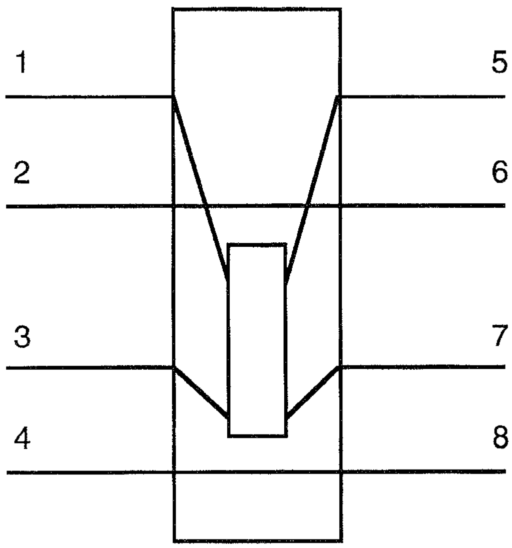

Fig 6. A 4+4 port network, consisting of a directional coupler and two through connections.

$$\times \begin{bmatrix} 0.9820 & 0 & 0.1890 & 0 \\ 0 & 1.0000 & 0 & 0 \\ -0.1890 & 0 & 0.9820 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & -1.0000 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$\times \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0.3625 & -0.9320 \\ 0 & 0 & -0.9320 & 0.3625 \end{bmatrix}. \quad (19)$$

Note that the identity matrices are implemented as simple straight through connections and therefore not shown in Fig. 7.

The scattering parameters of each of the six directional couplers are embedded in the appropriate submatrices of (19) (see Fig. 6.) In Fig. 7, the six directional couplers of Net A are labeled $A1$ through $A6$, and the directional couplers of network B are labeled $B1$ through $B6$. ($B3$ and $B5$ are missing since they correspond to identity matrices in (19).)

When the impedance specification has centro-symmetry [8], as in this example, one can reduce the number of directional couplers used in the realization. To demonstrate this point, the impedance of (10) is realized using only six couplers, i.e., one half of those required in an arbitrary 4×4 matrix lacking centrosymmetry.

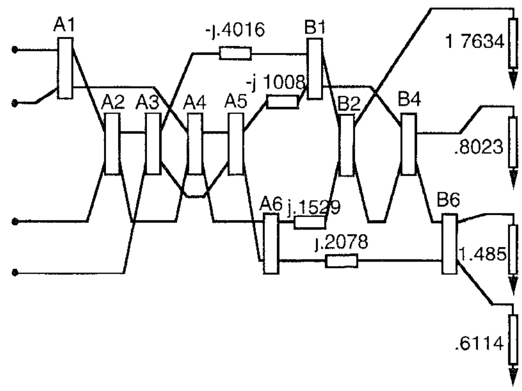

Fig 7 A realization of the coupling network whose admittance at ports one–four is (9)

The eigenvector matrix A was factored as

$$\begin{bmatrix} 0.4153 & -0.5723 & -0.4268 & -0.5638 \\ 0.5723 & 0.4153 & -0.5638 & 0.4268 \\ 0.5723 & 0.4153 & 0.5638 & -0.4268 \\ 0.4153 & -0.5723 & 0.4268 & 0.5638 \end{bmatrix} = A = A1 \times A2 \times A3 \times A4$$

where

$$A1 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0.7071 & -0.7071 & 0 \\ 0 & 0.7071 & 0.7071 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$A2 = \begin{bmatrix} 0.7071 & 0 & 0 & -0.7071 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0.7071 & 0 & 0 & 0.7071 \end{bmatrix}$$
$$A3 = \begin{bmatrix} 0.5873 & -0.8094 & 0 & 0 \\ 0.8094 & 0.5873 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$A4 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0.7973 & -0.6035 \\ 0 & 0 & 0.6035 & 0.7973 \end{bmatrix}.$$

The eigenvectors and eigenvalues of $A^t R_{\text{in}} A$ were found, and the matrix of eigenvectors B was factored as

$$\begin{bmatrix} 0.9820 & -0.1890 & 0 & 0 \\ 0.1890 & 0.9820 & 0 & 0 \\ 0 & 0 & 0.9320 & -0.3625 \\ 0 & 0 & 0.3625 & 0.9320 \end{bmatrix} = B = B1 \times B2.$$

where

$$B1 = \begin{bmatrix} 0.9820 & -0.1890 & 0 & 0 \\ 0.1890 & 0.9820 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$
$$B2 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0.9320 & -0.3625 \\ 0 & 0 & 0.3625 & 0.9320 \end{bmatrix}.$$

The corresponding reduced design is shown in Fig. 8.

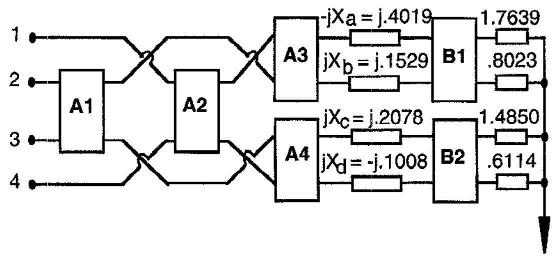

Fig. 8. Realization of Fig. 7 coupling network exploiting symmetry.

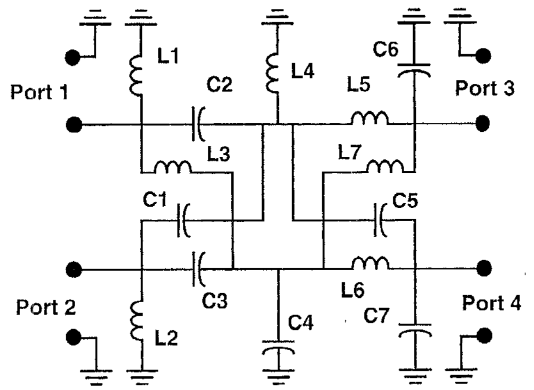

Fig. 9. Schematic diagram of the four-port couplers used in Fig. 8.

TABLE I
ELEMENT VALUES FOR THE FOUR-PART COUPLERS. CAPACITANCE IN pF, INDUCTANCE IN nH

	A1, A2	A3	A4	B1	B2
L1	0.980	1.215	0.843	0.589	0.664
L2	0.406	0.394	0.419	0.487	0.455
L3, L7	1.39	1.17	1.67	5.59	2.88
C1, C5	2.03	2.41	1.69	0.503	0.978
C2, C3	4.90	4.73	5.03	5.28	5.21
L4	0.693	0.584	0.833	2.80	1.44
C4	4.06	4.82	3.38	1.01	1.96
L5, L6	0.574	0.596	0.560	0.533	0.540
C6	6.93	7.14	6.72	5.78	6.19
C7	2.87	2.32	3.34	4.78	4.24

V. CIRCUIT SIMULATION AND DESIGN VERIFICATION

The block diagram of Fig. 8 can be realized by designing the six four-port couplers, $A1, A2, A3, A4, B1$, and $B2$.

To demonstrate the validity of the design procedure, the methods of [5] were followed to design the six directional couplers for operation at the center frequency of 3.0 GHz. The couplers were designed with lumped rather than distributed elements in order to insure the smallest possible physical size for the overall circuit. A characteristic impedance of 10 ohms was chosen for the circuit. This allows for practical values of lumped inductors and capacitors. By altering the feed point of the antenna slots, the antenna can be matched to the coupler at 3.0 GHz. The 50-ohm impedance of the source circuitry can be matched to the necessary terminating resistance of the circuit with quarter wave matching transformers. Each coupler is realized as shown by the schematic diagram of Fig. 9, and the component values for the couplers are given in Table I.

Note that Fig. 8 shows impedances normalized to one ohm, whereas element values listed in Table I and below assume a ten ohm characteristic impedance. Element values for the series admittances of Fig. 8 from top to bottom.

X_a is capacitive with a value of 13.2 pF.
X_b is inductive with a value of 81.1 pH.
X_c is inductive with a value of 110 pH.
X_d is capacitive with a value of 52.6 pF.

The program TOUCHSTONE was used to determine the frequency characteristics of the S-matrix of the lossless eight port network consisting of the six four-port couplers and the four reactances.

The admittance of the previously mentioned slot array antenna was calculated over a 10% bandwidth around the center frequency, using a separate electromagnetic simulation program. From this data the S-matrix was computed.

To investigate the performance of this coupling network in a transmitting mode, we consider a network connection as shown in Fig. 2. The excitation can be represented as four isolated voltage sources, each with a 50-Ω series resistance. In the simulation, we assume that the impedance transformers used to match the excitation characteristic impedance to the coupler circuitry have sufficient bandwidth that their contribution to the frequency response of the circuit is negligible.

Note that a perfect coupling network would extract the maximum power possible from each of the sources, which in this case would be $\frac{V_{\mathrm{RMS}}^2}{4Z_0}$ watts, where V_{RMS} is the rms voltage of the source and $Z_0 = 50$ ohms. The coupling network has been designed for a perfect match at 3 GHz, but will develop mismatch as the frequency varies about this point. To quantify the bandwidth performance of the coupling network, we considered five different voltage excitation vectors. These excitations are: each of the four-ports being driven separately by a one volt source and all four-ports driven together by four co-phasal one-half V sources, all in phase.

As Fig. 10 shows, the variation in power flow over a 10% bandwidth is different for each of the five excitations, ranging from less than 1–5dB. A future study is planned to investigate the sensitivity of the power flow to variations in the component values of the couplers.

VI. CONCLUSION

The paper develops a new algorithm for the design of lossless multiport matching networks based upon repeated applications of Givens rotations. The design technique parallels the procedure proposed by Carlin and Youla, but differs in its implementation by avoiding multiport transformers, which are impractical at microwave frequencies.

To illustrate the procedure, a coupling network for a four-element antenna array has been designed with six directional couplers, in order to maximize the power flow from four isolated transmitters into the antenna. Lumped elements were used in the design of the couplers. To validate the design, the antenna-coupler system has been simulated and its performance assessed over a 10% bandwidth. We expect the impressive bandwidth performance of our example to be the case with most designs obtained by this novel design methodology.

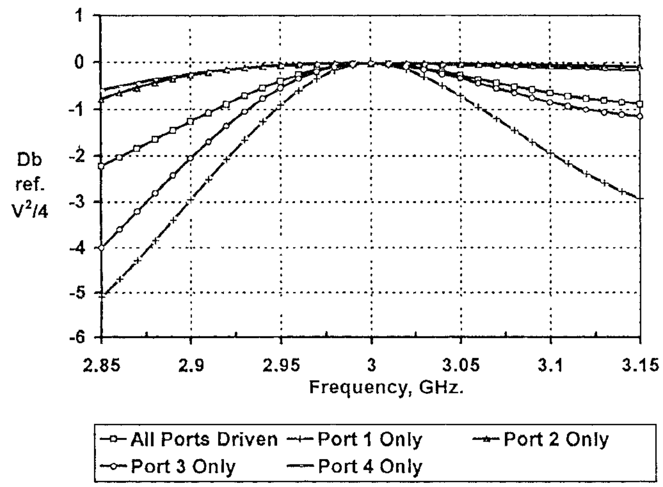

Fig. 10. Power from various source distributions.

APPENDIX
PROOF THAT NO MORE THAN $N(N-1)/2$ DIRECTIONAL COUPLERS ARE NEEDED TO REALIZE EACH OF THE $2N$ BY $2N$ NETWORKS A AND B

It will be shown that interconnections of networks of directional couplers, each with two input ports and two output ports, described by S-matrices of the form

$$S = \begin{bmatrix} 0 & 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & 0 & -\sin(\varphi) & \cos(\varphi) \\ \cos(\varphi) & -\sin(\varphi) & 0 & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 & 0 \end{bmatrix} \quad \text{(A1)}$$

can produce a $2N$-Port network with S-matrices of the form needed for networks A and B.

S_A and S_B, describing networks A and B, respectively, will be realized as cascades of $N+N$ port networks, an example of which is shown in Fig. 6, consisting of one $2+2$ port directional coupler and $(N-2)+(N-2)$ through connections.

The S-matrix of the network of Fig. 6 is of the form of (1). The A-matrix is diagonal for the row and columns where the ports are through, and has the entries of (A1) in the rows and columns corresponding to the directional coupler. For the example of Fig. 6, as in (A2) shown at the bottom of the page. Since the $N+N$ port networks have an S-matrix of the form of (1), the S-matrix of several such networks in cascade can be shown to be

$$S_{\text{total}} = \begin{bmatrix} 0 & A_{\text{total}} \\ A^t_{\text{total}} & 0 \end{bmatrix}. \quad \text{(A3)}$$

Where A_{total} is the product of individual A matrices of the $N+N$ port networks

$$A_{\text{total}} = A_1 A_2 \cdots A_n. \quad \text{(A4)}$$

Our design strategy is to find a decomposition of the upper right-hand quadrant of S_A or S_B into a product of matrices that are like the upper right-hand quadrant of (A2).

In the following, we provide this decomposition, by showing that, for an S-matrix of the form in (A3), the N by N real orthogonal matrix A_{total} may be expressed as a product of $N(N-1)/2$ matrices, each of which only involves two coordinate axes.

Consider the two-dimensional (2-D) unit vector in the x-y plane, $\hat{\rho}(\theta) = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$, where $\hat{x}$ and $\hat{y}$ are unit vectors along the x- and y-axes. Such an arbitrary unit vector may be rotated to lie along the x-axis with the orthogonal transformation matrix $R(-\theta)$, where

$$R(-\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

i.e.,

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}. \quad \text{(A5)}$$

Now suppose we have an N-dimensional unit $\vec{e}$ vector and select an arbitrary 2-D subspace containing a nonnull projection of $\vec{e}$. Applying a matrix of the form shown in (A1), one can rotate the projection of $\vec{e}$ to lie along either axis of the subspace. By repeated rotations (for a total of at most $N-1$), one can transform an arbitrary vector to lie along any of the N axes in the space.

Now consider the N by N real orthogonal matrix A. Since it is orthogonal, A may be expressed in terms of an orthonormal set of column vectors, i.e., $A = (\vec{e}_1 \vec{e}_2 \cdots \vec{e}_{N-1} \vec{e}_N)$. As has been shown, the first column vector may be rotated to the first axis (a one in the first row and zeros elsewhere) by multiplication with $N-1$ rotation matrices $G(1, m, -\theta_{1m})$, where $G(1, m, -\theta_{1m})$ is an N by N unit matrix, except for the $1, m$ subspace, which has the form of (A5). These operations are known as Givens rotations [1]. Application of $G(1, m, -\theta_{1m})$ annihilates the mth element in the first column of A.

Hence we have[1] $\vec{e}'_1 = \prod_{m=N}^{2} G(1, m, -\theta_{1m})\vec{e}_1$ with $\vec{e}'_1$ the unit vector along the first axis, having a one in the first row and zeros elsewhere. But the product of two orthogonal

[1] Note that the matrices are multiplied from left to right, starting with the first index of the product sign.

$$S_{4+4} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cos(\varphi) & 0 & -\sin(\varphi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin(\varphi) & 0 & \cos(\varphi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad \text{(A2)}$$

matrices is also orthogonal, so that the transformed vectors $\vec{e}_i' = \prod_{m=N}^{2} G(1, m, -\theta_{1m})\vec{e}_i$ form an orthonormal set.

Since only $\vec{e}'_1$ has a nonzero entry in the first row, the transformed A has the form

$$G_1 A = \begin{pmatrix} 1 & 0 & \cdot & 0 \\ 0 & & & \\ \cdot & & A_1 & \\ 0 & & & \end{pmatrix} \quad \text{(A6)}$$

where

$$G_1 = \prod_{m=N}^{2} G(1, m, -\theta_{1m}).$$

Repeating the process on the $N - 1$ dimensional subspace corresponding to A_1 with at most $N - 2$ rotations one can rotate $\vec{e}'_2$ to lie along the second axis, so that

$$G_2 G_1 A = \begin{pmatrix} 1 & 0 & \cdot & 0 \\ 0 & 1 & \cdot & 0 \\ 0 & 0 & & \\ \cdot & \cdot & A_2 & \\ 0 & 0 & & \end{pmatrix} \quad \text{(A7)}$$

where

$$G_2 = \prod_{m=N}^{3} G(2, m, -\theta_{2m}).$$

Continuing through the remaining subspaces, one finally obtains

$$\left(\prod_{i=N-1}^{1} G_i\right) A = \prod_{i=N-1}^{1} \prod_{m=N}^{i+1} G(i, m, -\theta_{im}) A = I. \quad \text{(A8)}$$

But $R(\theta)R(-\theta) = I$, so that (A8) handily inverts to the form

$$A = \prod_{i=1}^{N-1} \prod_{m=i+1}^{N} G(i, m, \theta_{im})$$

which is the desired result.

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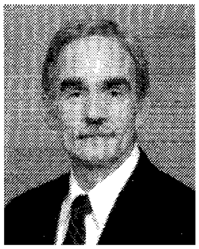

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